Tracking the galactic dark matter halo by modeling the vertical HI scale height data

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Plan of the talk

- A brief review of disc galaxies
- Rotation curve & HI vertical scale height data as probes to the galactic DM halo
- Our galactic disc model of gravitationally-coupled stars & gas
- Formulation & solution of the equations; Fits to observed data
- Application to M31
- Application to UGC 7321
- Summary
- Future work
A brief review of disc galaxies

A spiral galaxy consists of a disc of stars and gas [visible] embedded in a gigantic envelope of the dark matter halo [invisible] as is deduced from the flat galactic rotation curve assuming Newtonian dynamics.

**Disc**
- Stars constitute 90% whereas gas 10% of the total mass of the disc.
- The stellar disc scale length ($R_D$) is of the order of a few kpc. HI disc is extended beyond a few times $R_D$.
- Stars have an order of magnitude higher velocity dispersion (18 km/s) compared to gas (5-8 km/s).
- The disc is rotation-supported in the plane of the galaxy. Perpendicular to the plane, it is pressure-supported.

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HI Gas

[Mg = 0.1 Ms]

σ_g = 8 kms⁻¹

σ_s = 18 kms⁻¹ R_D

3-4 R_D

Dark Matter

Stars [Ms = 0.1 M_G]

Bulge 250 pc 350 pc

1 kpc
Rotation curve & HI vertical scale height data as probes to the galactic DM halo

Rotational velocity at any radius depends on the total mass enclosed within that radius. So it is weakly dependent on the shape of the spheroidal halo.

HI scale height depends on the net downward gravitational pull of the disc, which in turn depends on the shape of the halo. A more flattened halo produces a larger downward constraining force, and vice versa.

So the rotation curve & the HI vertical scale height data complement each other in obtaining the DM halo properties uniquely.
• Each of the 2 disc components (stars + HI gas) is in the form of a thin, axisymmetric disc with an isothermal velocity dispersion. The discs are coplanar and embedded into each other.

• The discs are gravitationally coupled to each other, and subjected to the force-field of a dark matter halo of known density profile with four free parameters. (de Zeeuw & Pfenniger 1988).
\[ \rho ( R, z ) = \frac{\rho_0}{\left( 1 + \frac{m^2}{R_c^2} \right)^p} \]

\[ m^2 = R^2 + z^2, \]

\[ \rho_0 - \text{core-density,} \]
\[ R_c - \text{core-radius,} \]
\[ p - \text{density index,} \]
\[ q - c/a - \text{axis ratio.} \]

Our aim is to find the best-fit values of these parameters.
Formulation & solution of equations

Formulation

The vertical structure of the gas at any radius is determined by the balance between the upward hydrostatic pressure and the total downward gravitational pull.

The equation of hydrostatic equilibrium in the z direction for each of the two disc components is given by

$$\frac{< (v_z)_i^2 >}{\rho_i} \frac{d \rho_i}{dz} = - \left( \frac{d \psi_s}{dz} \right) - \left( \frac{d \psi_{HI}}{dz} \right) - \left( \frac{d \psi_{DM}}{dz} \right) \quad \ldots \ldots (1)$$

$i = \text{stars, HI}$
The joint Poisson equation for disc + halo assuming axi-symmetry and a flat rotation curve is given by

\[
\frac{d^2 \psi_s}{dz^2} + \frac{d^2 \psi_{HI}}{dz^2} + \frac{d^2 \psi_{DM}}{dz^2} = 4\pi G (\rho_s + \rho_{HI} + \rho_{DM})
\] ......(2)

Combining (1) & (2),

\[
\frac{d^2 \rho_i}{dz^2} = \frac{\rho_i}{\langle (v_z)_i \rangle^2} \left[ -4\pi G (\rho_s + \rho_{HI} + \rho_{DM}) \right] + \frac{1}{\rho_i} \left( \frac{d\rho_i}{dz} \right)^2
\] ....(3)

which represents two coupled, second-order, ordinary differential equations involving stellar and gas density respectively.
Solution of the equation

\[
\frac{d^2 \rho_i}{dz^2} = \frac{\rho_i}{<(v_z)_i^2>} \left[ -4\pi G (\rho_s + \rho_{HI} + \rho_{DM}) \right] + \frac{1}{\rho_i} \left( \frac{d\rho_i}{dz} \right)^2
\]

These are solved numerically as an initial value problem using Fourth order, Runge-kutta method of integration in an iterative fashion.

**Boundary conditions**

At \( z = 0 \)

\[
\rho_i = (\rho_i)_0 \quad \text{determined indirectly by trial and error method from} \quad \Sigma_R
\]

\[
\left( \frac{d\rho}{dz} \right)_{z=0} = 0
\]

At each \( R \), solution of the above equations gives \( \rho_{HI}(z) \), and thereby the scale height at \( R \), as a function of the free parameters of \( \rho_{DM} \)
Scanning the 3D parameter grid

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Range</th>
<th>Step-size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0 , (\text{M}_{\odot} \text{pc}^{-3})$</td>
<td>0.001 – 0.15</td>
<td>0.001</td>
</tr>
<tr>
<td>$\text{Rc} , (\text{R}_{\odot})$</td>
<td>1 – 5</td>
<td>0.1</td>
</tr>
<tr>
<td>$p$</td>
<td>1 - 2</td>
<td>0.5</td>
</tr>
<tr>
<td>$q$</td>
<td>0.1 - 1</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Fits to observed data

Rotation Curve Fit

We fit the theoretical rotation curve for each such grid point to the observed one and obtain the chi-square of the fit. We choose the grid points having chi-square values of the order of the number of data points in the observed curve, to apply the next constraint.

HI scale height fit

For each of the above grid points, we obtain the theoretical HI scale height curve and fit it to the observed curve beyond three disc scale lengths from where the disc gravity gives way to the DM halo gravity.

The minimum of the chi-square determines the best fit DM halo parameters.
Applicati**on to Andromeda (M31)**

1. The highly flattened shape of the DM halo holds important implications for galaxy formation and evolution.

2. Cosmological simulations (Bailin & Steinmetz 2005, Bett et al. 2007) give a range, and 0.4 is at the most oblate end (not typical!)

3. This implies either M31 is an unusual galaxy or more likely, simulations need to include additional physics like effects of baryons. Confirmed by Read et al.(2008), Debattista et al. (2008)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best-fit values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$ (M$_{\odot}$pc$^{-3}$)</td>
<td>0.011</td>
</tr>
<tr>
<td>$R_c$ (Kpc)</td>
<td>21</td>
</tr>
<tr>
<td>$p$</td>
<td>1 (isothermal)</td>
</tr>
<tr>
<td>$q$</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Best-fit case for a spherical (q=1) halo

Calculated scale heights too large at large radii --- Spherical halo ruled out.
Try Flattened halo --- higher mid-plane density --- expect lower scale heights

HI scale height data:
Braun 1991
Best fit case ($q = 0.4$) for a flattened dark matter halo

HI scale height data: 

Braun 1991
\( \chi^2 \) for best-fit case for each \( q \) versus axis ratio \( q \)

Dark matter halo with \( q=0.4 \) has lowest \( \chi^2 \) and is clearly distinguished from spherical case.
DM Halo dominates beyond $R = 25$ kpc

Halo constrained better if data available at $R > 25$ kpc.
Application to UGC 7321

(Banerjee, Matthews & Jog 2009, submitted)

Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best-fit values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_0$ (M$_{\text{sun}}$pc$^{-3}$)</td>
<td>0.039 – 0.057</td>
</tr>
<tr>
<td>$R_c$ (Kpc)</td>
<td>2.5 – 2.9</td>
</tr>
<tr>
<td>p</td>
<td>1 (isothermal)</td>
</tr>
<tr>
<td>q</td>
<td>1 (spherical)</td>
</tr>
</tbody>
</table>

$R_c \sim R_D = 2.1 \text{ Kpc.}$

*This shows that this superthin galaxy is dark matter dominated even at small radii!*
The best-fit requires high gas dispersion. This can explain why there is little star formation these galaxies (hence LSB).
Interestingly, the DM dominates already by 4 kpc ($2 R_D$). This is clear evidence that LSBs are dark-matter dominated.
Summary

This is quite a versatile method which can help to determine

- Core density
- Core radius
- Density index
- Shape of the dark matter halo

provided we have the outer galactic HI scale height data which continues to be a difficult and challenging task even to this day (Sancisi & Allen 1979)
Future Work

1. To reinvestigate the dark matter halo of M31 with Prof. Robert Braun (Australia), who now has an improved HI data for M31, from which $v_{z,\text{HI}}$ as a function of $R$ & $z$ can be deduced.

2. To apply our model to study the DM halos of the FIGGS (Faint Irregular GMRT Galaxy Survey) galaxies with Prof. Jayaram Chengalur (India)
University Observatory
Munich
Danke!